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STRATEGIC INCENTIVES FOR KEEPING ONE SET OF BOOKS UNDER THE ARM'S LENGTH PRINCIPLE*

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Abstract

The OECD's recommendation that transfer prices between multinational enterprises and their subsidiaries be consistent with the Arm's Length Principle (ALP) for tax purposes does not restrict internal pricing policies. However, we show that under imperfect competition firms may choose to keep one set of books (i.e., to set transfer prices consistent with the ALP), as a way of softening competition in the external market. As a result, firms' profits are greater, and the surplus is smaller, than under vertical integration. In contrast, when firms keep two sets of books (i.e., their transfer prices differ from those used for tax purposes), competition intensifies in both markets relative to vertical integration.

Keywords: Transfer Pricing Regulation, Arm's Length Principle, Imperfect Competition, Vertical Separation.

JEL Classification: L1, L5, H2.

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1 Introduction

Transfer pricing policies of multinational firms have important implications since exports and imports from related parties are a dominant share of the trade flows – see Bernard, Jensen and Schott (2009). Transfer prices serve the purpose of both allocating costs to subsidiaries and determining the tax liabilities of parents and subsidiaries. Policy makers are aware of the possible use of transfer prices as a device for shifting profits into low tax jurisdictions, and tend to follow the OECD Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations, which recommend that, for tax purposes, internal pricing policies be consistent with the *Arm's Length Principle* (ALP); i.e., that transfer prices between companies of multinational enterprises be established, for tax purposes, on a market value basis, thus comparable to transactions between independent (unrelated) parties -see [23]. Multinational firms must therefore choose whether to use a single transfer price for both internal transactions and tax purposes (i.e., keep *one set of books*), or set internal transfer prices different from those used for tax purposes (i.e., keep *two sets of books*).

Under vertical integration the choice between keeping one or two sets of books is irrelevant. However, under delegation this choice may be relevant since managerial incentives may not internalize properly the impact of decisions on the firms' consolidated profits, even when tax rates are equal across jurisdictions. Thus, the impact of alternative accounting policies may depend on the market structure. In a monopoly setting, Baldenius, Melumad and Reichelstein (2004) show that keeping two sets of books allows firms to deal with conflicting managerial objectives. Under price competition, Göx (2000), and Dürr and Göx (2011) find that firms may benefit from strategically using the same transfer price for tax and managerial purposes, concluding that the ALP may provide a rationale for vertical separation. (The literature has established that in the absence of the ALP vertical separation intensifies or alleviates competition depending on whether firms compete in quantities or prices – see Vickers (1985), Fershtman and Judd (1987), Sklivas (1987), Alles and Datar (1998).)

In this paper, we study the accounting policies that arise when transfer prices are consistent with the ALP for tax purposes, as well as the properties of the ensuing market equilibria, under vertical separation, imperfect competition and quantity setting. Quantity competition provides a reduced form model for the analysis of more

complex forms of imperfect competition; e.g., capacity choice followed by some kind of price competition – see Kreps and Scheinkman (1983) and Moreno and Ubeda (2006). Following the literature, we assume that parents maximize consolidated profits, while subsidiaries maximize their own profits – see Gal-Or (1993). Since changing a firm’s accounting policy typically involves high administrative and consulting costs, in our setting the choice of accounting policy serves as a commitment device – see Göx (2000), Arya and Mittendorf (2008) and Dürr and Göx (2011). (The literature has studied other commitment instruments such as distorting managerial compensation, e.g., Fershtman and Judd (1987), Sklivas (1987), sinking capacity investments, e.g., Dixit (1980), Spence (1977), building inventories, e.g., Ware (1985), limiting information acquisition, e.g., Einy, Moreno and Shitovitz (2002), Gal-Or (1988), or using cost allocation rules, e.g., Gal-Or (1993), Hughes and Kao (1998).) Moreover, since accounting policies tend to be public, we assume that they are observed by competing firms before making output and transfer pricing decisions. (Accounting policies are disclosed in management discussions and annual reports, and are reported to securities and exchange commissions, tax authorities, etc.)

In our framework there are two markets, which we refer to as the Latin (home) market and the Greek (external) market. There are two firms engaging in Cournot competition in the Latin market. These firms have subsidiaries, which in turn engage in Cournot competition in the Greek market. Parent firms simultaneously choose their accounting policies. Upon observing accounting policy choices, parents simultaneously make output and, when relevant, internal transfer price decisions. Competition in the Latin market provides a reference price on comparable market transactions, and hence determines the firms’ tax bills. Subsidiaries, upon observing accounting policies, outputs and internal transfer pricing decisions, simultaneously make output decisions. Thus, competing firms face a three stage non-cooperative game of complete information. A subgame perfect equilibrium (SPE henceforth) of this game identifies the firms’ accounting policies as well as their outputs in the Latin and Greek market. We study the properties of SPE.

Using backward induction, we identify the SPE that may arise in our setting. Given parents accounting policies, outputs, and transfer pricing decisions, which determine the subsidiaries’ costs, subsidiaries compete a la Cournot. It is therefore

straightforward to identify the ensuing equilibrium in the Greek market. We proceed to study the equilibria that arise in subgames identified by parents accounting policy decisions. Using these results, we determine the accounting policies that are sustained by SPE.

In a subgame in which both parents keep one set of books (i.e., the good is transferred to subsidiaries at the price it is sold in the Latin market), a parent's output decision must internalize its impact on the transfer price of its subsidiary (as well as its subsidiary's rival). Thus, parents' efforts to alleviate double marginalization intensifies competition in the Latin market. As a result, in equilibrium the output served to the Latin (Greek) is larger (smaller), and hence the surplus realized is larger (smaller), than under vertical integration. However, we show that firms' consolidated profits are greater, although the total surplus is smaller, than under vertical integration. Thus, keeping one set of books serves parents as an instrument to soften competition in the Greek market, and ultimately leads to larger consolidated profits. Hence the adoption of the ALP, when it leads firms to keep one set of books, provides a rationale for vertical separation. (This conclusion stands in sharp contrast to the results of Göx (2000) and Dürr and Göx (2011), who find that with quantity competition vertical separation intensifies competition.) Note that when both firms keep one set of books adopting the ALP as a guideline for regulating transfer prices has negative welfare implications for the external country, as the surplus in the Greek market decreases. (Interestingly, adopting the ALP does not achieve the objective of protecting the external country's tax base either – see Lemus and Moreno (2019).)

In a subgame in which both parents keep two sets of books, i.e., transfer prices differ from those used for tax purposes, internal transfer prices give parents an instrument to gain a sort of Stackelberg advantage in the external market. As a result, competition intensifies in the Greek market. Interestingly, the ALP creates a subtle link between the two markets, which intensifies competition in the Latin market also: since the transfer price for tax purposes is the price in the Latin market, each parent can improve the competitive advantage of its subsidiary by increasing the tax liability of its subsidiary's rival, which can be achieved by increasing output in the Latin market, thus reducing the price. As a result, the output in both markets are greater, and hence the surplus is larger, than under vertical integration. Therefore, adopting

the ALP improves market efficiency when firms keep two sets of books.

In a subgame in which parents accounting policies are asymmetric, i.e., one parent keeps one set of books and the other keeps two sets of books, the subsidiary of the firm keeping two sets of books becomes dominant in the external market, while the parent using one set of books becomes the dominant producer in the home market. We show that in both markets the output and the surplus are larger, while the sum of the firms' profits is smaller, than under vertical integration. Thus, when at least one firm choose to keep one set of books, the adoption of the ALP leads to a surplus increase in both markets.

With these results in hand, we study the accounting policies that are sustained by SPE of the dynamic game firms face. (We restrict attention to pure strategy equilibria.) Assuming that play following parents' accounting policy decisions forms a SPE, we show that parents' interaction at this first stage is described by a simple symmetric two-action static game G . A Nash equilibrium of G corresponds to a SPE of the dynamic game.

Depending of the parameters of the model, which in our framework are the tax rate common to both markets, and the size of the Latin market relative to the Greek market, G is either a prisoners' dilemma game, a game of chicken, a coordination game, or a cooperation game. When G is a prisoners' dilemma game, the unique SPE involves both firms keeping two sets of books. When G is a game of chicken, the two (pure strategy) SPE of the game involve firms using asymmetric accounting policies. When G is a coordination game, there are two SPE: one in which both firms keep one set of books, and another one in which both firms keep two sets of books. Finally, When G is a cooperation game, the unique SPE involves both firms keeping one set of books.

When G is either a coordination or a cooperation game the welfare consequences of adopting the ALP are negative. It is worth noting that from the firms' point of view the SPE where firms keep one set of books Pareto dominates that in which both firms keep two sets of books. Thus, even when G is a coordination game, any equilibrium concept that accounts for firms' communication opportunities, e.g., Ferreira (1996)'s communication equilibrium, will select this equilibrium as the more likely. When G is either a prisoners' dilemma game or a game of chicken, the welfare consequences

of adopting the ALP are positive. It is therefore useful to explore the parameter constellations that give rise to the different types of games.

We show that all four types of games may arise. A coordination or a cooperation game arises when the relative size of the Latin market is small, or when both the relative size of the Latin market and the tax rate are quite large. In these cases, the incentives for a firm to switch to keeping two sets of books when the rival keeps one set are small because when both firms keep one set of books the equilibrium in the Greek market is close to the monopoly outcome; hence firms' profits are close to half of the monopoly profits. A deviating firm stands to make at most have of the monopoly profits (if it manages to gain a short of Stackelberg leader position), while it loses entirely its position in the Latin market. Thus, such deviations are not profitable.

For intermediate values of the relative size of the Latin market and the tax rate, a prisoners' dilemma game arises, whereas for larger values of these parameters a game of chicken arises. For the parameter constellations with these properties, a firm choosing to keep one set of books is in a weak position given the intensity of competition in both the home and the external markets. See Figure 1 for a more precise description of the parameter constellations supporting alternative accounting policies.

While adopting the ALP is inconsequential under perfect competition, or when firms are vertically integrated, under vertical separation, its neutrality is lost: the adoption of the ALP significantly affect firms' behavior, and hence market outcomes.

The paper is organized as follows. We introduce the basic setup in Section 2. In Section 3 we study the equilibria of the subgames given the firms' choice of accounting policies. In Section 4 we study the choice of accounting policies. We conclude in Section 5. The proofs of our results are relegated to the Appendix.

2 Model and Preliminaries

A good is sold in two markets, the *Latin* market and the *Greek* market, in which the inverse demand functions are $d(q) = \max\{0, 1 - bq\}$ and $\delta(\chi) = \max\{0, 1 - \beta\chi\}$, respectively. In these formulae, q and χ are the units of good demanded in the

Latin and Greek markets at prices $p = d(q)$ and $\pi = \delta(\chi)$, respectively, and b and β are positive real numbers. There are two firms producing the good with the same constant marginal cost, which is assumed to be zero without loss of generality.

Assuming linear demands and identical maximum willingness to pay in both markets reduces notation, simplifies the analysis, and facilitates interpreting the results. (The equality of the maximum willingness to pay implicitly assumes that the range of per capita income are similar in both markets.) A measure of the size of the Latin (Greek) market is provided by the demand at price zero, $1/b$ ($1/\beta$). The smaller is b (β), the larger is the Latin (Greek) market. The parameter $s := \beta/b$ is thus a proxy for the size of Latin market relative to that of the Greek market. Also, we denote by $t, \tau \in (0, 1)$ the tax rates on profits in the Latin and Greek markets, respectively.

Under vertical integration and Cournot competition, in equilibrium the output of each firm, the price, and the surplus in the Latin and Greek markets are readily calculated as

$$(q_{VI}^*, p_{VI}^*, S_{VI}^*) = \left(\frac{1}{3b}, \frac{1}{3}, \frac{4}{9b} \right)$$

and

$$(\chi_{VI}^*, \pi_{VI}^*, \Sigma_{VI}^*) = \left(\frac{1}{3\beta}, \frac{1}{3}, \frac{4}{9\beta} \right),$$

respectively. Note that the tax rates do not affect market outcomes. Each firm's *consolidated profit* (i.e., its total revenue minus its total tax bill) is

$$C_{VI}^* = \frac{1-t}{9b} + \frac{1-\tau}{9\beta}.$$

We consider a setting of vertical separation in which firms engage in Cournot competition in the Latin market, and have *subsidiaries* which in turn engage in Cournot competition in the Greek market. Each *parent* firm seeks to maximize its consolidated profit, while each subsidiary seeks to maximize its *own profit* (i.e., its revenue, minus the total transfer paid to the parent, minus its tax bill). Also, we assume throughout that transfer prices must be consistent with the ALP for tax purposes, i.e., if a parent and its subsidiary serve q and χ units of output to the Latin and Greek markets at prices p and π , respectively, then the parent's tax bill is $tp(q + \chi)$, and the subsidiary's tax bill is $\tau(\pi - p)\chi$. Hence consolidated profit is

$$\begin{aligned} C(p, q, \pi, \chi) &= pq + \pi\chi - tp(q + \chi) - \tau(\pi - p)\chi \\ &= (1-t)pq + (1-\tau)\pi\chi + (\tau-t)p\chi. \end{aligned}$$

Further, if the parent uses p as the transfer price, a situation we refer to as *keeping one set of books*, then its subsidiary's profit is

$$\Pi_I(p, \pi, \chi) = (1 - \tau)(\pi - p)\chi, \quad (1)$$

whereas if the parent uses an internal transfer price r that differs from p , a situation we refer to as *keeping two sets of books*, then the subsidiary's profit is

$$\Pi_{II}(p, r, \pi, \chi) = (\pi - r)\chi - \tau(\pi - p)\chi. \quad (2)$$

In order to avoid results driven by differences in tax rates, we assume throughout that $t = \tau$, and we denote by τ the common tax rate. Under this assumption consolidated profit is

$$C(p, q, \pi, \chi) = (1 - \tau)(pq + \pi\chi). \quad (3)$$

Thus, maximizing consolidated profit amounts to maximizing revenue, i.e., the tax rate does not affect the parent's incentives. Moreover, the tax rate affects the subsidiary's incentives only when the parent keeps two sets of books.

In this setting of vertical separation, we will assume that parents first decide simultaneously their accounting policies, i.e., whether they keep one set of books or two sets of books. Then, upon observing the accounting policies chosen, parents simultaneously decide how much output to supply to the Latin market as well as their transfer prices. The subsidiaries decide how much output to serve to the Greek market upon observing the parents' decisions.

As argued in the introduction, assuming that firms commit to their accounting policies is reasonable if, for example, the costs associated with changing the accounting policy are sufficiently high. Göx (2000) notes that a new accounting policy usually requires substantial investments in developing or acquiring software and in training employees and/or hiring consultants. By choosing to keep one set of books, a parent commits to use the Latin market price as the transfer price per intrafirm transaction. Likewise, a parent that chooses to keep two sets of books allows itself the flexibility of using an internal transfer price different from the price in the Latin market.

Our objective is to identify the conditions under which alternative accounting policies can be sustained by subgame perfect equilibria (SPE henceforth), and to study the properties of the outcomes they generate. We restrict attention to SPE in pure strategies.

3 Equilibria with Alternative Accounting Policies

The parents' accounting policies, their decisions about the output they serve to the Latin market and, when relevant, their transfer prices, identify the subgame in which subsidiaries decide how much output to serve to the Greek market. In a SPE the subsidiaries' output decisions must form an equilibrium of the corresponding subgame. Moreover, in the subgame following firms' accounting policy decisions, parents' outputs and transfer pricing decisions (when they apply) must form an equilibrium when parents anticipate the ensuing equilibrium outcome in the Greek market following their decisions. We identify the market equilibria that arise under alternative accounting policies.

ONE SET OF BOOKS

Consider a SPE in which both parents keep *one set of books*, i.e., they commit to using the price in the Latin market p as the transfer price per intrafirm transaction. Thus, the constant marginal cost of both subsidiaries is p . Then the subsidiary's output at the Cournot equilibrium of the Greek market, $\chi_I(p)$, and the equilibrium price, $\pi_I(p)$, can be readily calculated as

$$\chi_I(p) = \frac{1 - p}{3\beta}, \quad (4)$$

and

$$\pi_I(p) = \delta(2\chi_I(p)) = \frac{1 + 2p}{3}. \quad (5)$$

A parent's output in the equilibrium of the subgame, q_I^* , maximizes consolidated profits (i.e., revenue), when the output of the competitor is q_I^* , and the subsidiaries outputs following the parents' decisions are $(\chi_I(p), \chi_I(p))$, where $p = d(q_I^* + q_I^*)$. That is, q_I^* solves the problem

$$\max_{q \in \mathbb{R}_+} C(d(q + q_I^*), q, \pi_I(d(q + q_I^*)), \chi_I(d(q + q_I^*))).$$

The first order condition for a solution to this problem identifies q_I^* . Thus, the equilibrium price in the Latin market is $p_I^* = d(2q_I^*)$. Substituting the value of p_I^* into the formulae for $\chi_I(p)$ and $\pi_I(p)$ given above, we obtain the subsidiaries' output, χ_I^* . Finally, the equilibrium price in the Greek market is $\pi_I^* = \delta(2\chi_I^*)$. The consolidated profits are readily calculated using equation (3).

We describe our results in Proposition 1. This description involves the function $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ given for $s \in \mathbb{R}_+$ by

$$f(s) := \frac{2 + 9s}{8 + 27s}.$$

Note that f is strictly increasing and satisfies $f(0) = 1/4$ and $\lim_{s \rightarrow \infty} f(s) = 1/3$.

Proposition 1. *In a SPE in which both parents keep one set of books the output of each parent is*

$$q_I^* = \frac{1 - f(s)}{2b} > \frac{1}{3b} = q_{VI}^*,$$

and the price in the Latin market is $p_I^ = f(s) < 1/3 = p_{VI}^*$, while the output of each subsidiary is*

$$\chi_I^* = \frac{1 - f(s)}{3\beta} < \frac{1}{3\beta} = \chi_{VI}^*,$$

and the price in the Greek market is

$$\pi_I^* = \frac{1 + 2f(s)}{3} > \frac{1}{3} = \pi_{VI}^*.$$

Moreover, the firms' consolidated profit C_I^ are greater, and the total surplus $S_I^* + \Sigma_I^*$ is smaller, than under vertical integration, i.e., $C_I^* > C_{VI}^*$ and $S_I^* + \Sigma_I^* < S_{VI}^* + \Sigma_{VI}^*$.*

When parents keep one set of books, they supply more output to the (home) Latin market, while their subsidiaries supply less output to the (external) Greek market, than under vertical integration. The intuition of this results is simple: parents' incentives to alleviate double marginalization leads then to increase their output in the Latin market above q_{VI} . Keeping one set of books, i.e. setting internal prices consistently with the ALP, serves as an instrument for parents to soften competition in the Greek market. The incentive to keep one set of books is sharper the larger the size of the Greek market relative to that of the Latin market, i.e., the smaller is s . Note that as s tends to zero, i.e., β approaches zero or b becomes arbitrarily large, the equilibrium of the Greek market approaches the monopoly outcome. (A related result of Choe and Matsushima (2013) shows that the ALP facilitates tacit collusion in dynamic imperfectly competitive markets.)

Of course, since the transfer price is determined by parents' outputs in the Latin market, competition in this market is more aggressive than under vertical integration.

Nevertheless, consolidated profits are greater than under vertical integration. Thus, this accounting policy may provide a rationale for vertical separation.

When parents keep one set of books, however, both the surplus in the Greek market and the total surplus are smaller than under vertical integration. This conclusion raises questions about the social benefits of adopting the ALP as a guideline for regulating transfer prices when this policy induces firms to keep one set of books. (In addition, Lemus and Moreno (2019) show that in this case the ALP fails to protect the external country tax base.)

TWO SETS OF BOOKS

Let us now consider a SPE in which both parents keep *two sets of books*. If the price in the Latin market is p , and parent i uses the transfer price r_i , while its competitor uses the transfer price r_{-i} , then subsidiaries compete a la Cournot maximizing their profit as given by the function Π_{II} . In an interior equilibrium of the Greek market following these decisions, subsidiary i supplies the output

$$\chi_{II}(p, r_i, r_{-i}) = \frac{1}{3\beta} + \frac{\tau p - 2r_i + r_{-i}}{3\beta(1 - \tau)}, \quad (6)$$

and the price in the Greek market is

$$\pi_{II}(p, r_i, r_{-i}) = \frac{1}{3} - \frac{2\tau p - r_i - r_{-i}}{3(1 - \tau)}. \quad (7)$$

Thus, a parent's output q_{II}^* and transfer price r_{II}^* solve the problem

$$\max_{(q, r) \in \mathbb{R}_+ \times \mathbb{R}} C(d(q + q_{II}^*), q, \pi_{II}(d(q + q_{II}^*), r, r_{II}^*), \chi_{II}(d(q + q_{II}^*), r, r_{II}^*)).$$

Solving the system of first order conditions for an interior solution to this problem we get (q_{II}^*, r_{II}^*) , and $p_{II}^* = d(2q_{II}^*)$. It turns out that if $s \leq g(\tau)$, where $g : [0, 1] \rightarrow \mathbb{R}$ is giving for $s \in \mathbb{R}_+$ by

$$g(\tau) = \frac{2\tau}{5(1 - \tau)},$$

then in equilibrium $p_{II}^* = 0$, and the parents' consolidated profit are independent of their output in the Latin market. In this case, r_{II}^* solves problem

$$\max_{r \in \mathbb{R}_+} C(0, q, \pi_{II}(0, r, r_{II}^*), \chi_{II}(0, r, r_{II}^*)).$$

In either case, substituting p_{II}^* and r_{II}^* in the formulae above we get χ_{II}^* and π_{II}^* . Consolidated profits are readily calculated using equation (3). We describe our results in Proposition 2.

Proposition 2. *In a SPE in which both parents keep two sets of books the output of each subsidiary is*

$$\chi_{II}^* = \frac{2}{5\beta} > \chi_{VI}^*,$$

and the price in the Greek market is $\pi_{II}^ = 1/5 < \pi_{VI}^*$. Further:*

(P2.1) If $s > g(\tau)$, then the output of each parent is

$$q_{II}^* = \frac{2s + g(\tau)}{6bs} > q_{VI}^*,$$

and the price in the Latin market is

$$p_{II}^* = \frac{s - g(\tau)}{3s} < p_{VI}^*.$$

(P2.2) If $s \leq g(\tau)$, then the price in the Latin market is $p_{II}^ = 0$, and parents serve the demand at this price.*

Moreover, in either case consolidated profits, C_{II}^ , are smaller than under vertical integration, i.e., $C_{II}^* < C_{VI}^*$, while the surpluses in both the Latin and Greek markets, S_{II}^* and Σ_{II}^* , respectively, are larger than under vertical integration, i.e., $S_{II}^* > S_{VI}^*$ and $\Sigma_{II}^* > \Sigma_{VI}^*$.*

When firms keep two sets of books, the ALP creates a subtle link between the Latin and Greek markets, which intensifies competition in both markets: because for tax purposes the transfer price is the price in the Latin market, each parent has an incentive to improve the competitive advantage of its subsidiary by increasing the output it supplies to the Latin market, thus reducing the price and increasing the tax liability of its subsidiary's rival. This incentive is not present when both firms keep one set of books, or when they are vertically integrated. In addition, each parent has incentives to reduce the internal transfer price in order to give a cost advantage to its subsidiary, which makes competition more aggressive in the Greek market as well.

These incentives give rise to an equilibrium with significantly better welfare properties than that arising when parents keep one set of books: in this scenario the output in both markets are greater (rather than smaller), and hence the surplus is larger than under vertical integration. Therefore, adopting the ALP as a guideline for regulating transfer prices improves market efficiency when firms keep two sets of books. On the other hand, consolidated profits are smaller than under vertical integration. Hence, when the adoption of the ALP give rise to an equilibrium in which

firms keep two sets of books, vertical separation is hardly an optimal organizational design.

ASYMMETRIC ACCOUNTING POLICIES

Let us consider now the case in which one parent (which we identify with the subindex I) keeps *one set of books*, while the other parent (II) keeps *two sets of books*. Consider a subgame in which the subsidiaries compete in quantities after observing the price in the Latin market, \hat{p} , and the transfer price chosen by Parent II , \hat{r}_{II} . Subsidiary I (the subsidiary of Parent I) supplies the output $\hat{\chi}_I$ that solves the problem

$$\max_{\chi \in \mathbb{R}_+} \Pi_I(\hat{p}, \delta(\chi + \hat{\chi}_{II}), \chi).$$

The solution to this problem gives us the *reaction function* of Subsidiary I to $\hat{\chi}_{II}$, given \hat{p} and \hat{r}_{II} . Likewise, Subsidiary II (the subsidiary of Parent II) supplies the output $\hat{\chi}_{II}$ that solves the problem

$$\max_{\chi \in \mathbb{R}_+} \Pi_{II}(\hat{p}, \hat{r}_{II}, \delta(\hat{\chi}_I + \chi), \chi).$$

The solution to this problem gives us the reaction function of Subsidiary II to $\hat{\chi}_I$, given \hat{p} and \hat{r}_{II} . The solution to the system of equations formed by these reaction functions is

$$\begin{aligned} \hat{\chi}_I(\hat{p}, \hat{r}_{II}) &= \max \left\{ \frac{1 - \hat{p}}{3\beta} - \frac{(\hat{p} - \hat{r}_{II})}{3\beta(1 - \tau)}, 0 \right\}, \\ \hat{\chi}_{II}(\hat{p}, \hat{r}_{II}) &= \max \left\{ \frac{1 - \hat{p}}{3\beta} + \frac{2(\hat{p} - \hat{r}_{II})}{3\beta(1 - \tau)}, 0 \right\}. \end{aligned}$$

Let us consider parents' decisions when that they anticipate that the subsidiaries decide the output they serve according the functions $(\hat{\chi}_I, \hat{\chi}_{II})$. Parent I supplies the output \hat{q}_I that solves the problem

$$\max_{q \in \mathbb{R}_+} C(d(q + \hat{q}_{II}), q, \hat{\pi}(d(q + \hat{q}_{II}), \hat{r}_{II}), \hat{\chi}_I(d(q + \hat{q}_{II}), \hat{r}_{II})).$$

The first order condition for a solution to this problem identifies Parent I 's reaction function to the output and the transfer price set by Parent II . Likewise, Parent II 's output and transfer price, $(\hat{q}_{II}, \hat{r}_{II})$, solves the problem

$$\max_{(q, r) \in \mathbb{R}_+ \times \mathbb{R}} C(d(q + \hat{q}_I), q, \hat{\pi}(d(q + \hat{q}_I), r), \hat{\chi}_{II}(d(q + \hat{q}_I), r)).$$

The system of first order conditions for a solution to this problem gives \hat{q}_{II} and \hat{r}_{II} as a function of \hat{q}_I .

In the proof of Proposition 3 (see the Appendix) we establish that there is no interior SPE sustaining firms' asymmetric accounting policies: the solution of the system of equations formed by the parents' reaction functions leads to a negative value for the output of Subsidiary I whenever $\tau < 1/2$. Since $\tau < 1/2$ in just about every country in the world, we proceed to identify a corner pure strategy SPE assuming that $\tau \in [0, 1/2]$. (According to Jahnsen and Pomerleau (2017), only two countries in the world, UAE and Comoros, have corporate tax rates above 50%. Auerbach *et al.* (2009) show that corporate tax rate are falling across G7 economies, and provide some evidence suggesting convergence of corporate tax rates to values between 30% and 40%.)

Also, we show in the proof of Proposition 3 that in a SPE where firms use asymmetric accounting policies the output of Parent II is zero, i.e., $\hat{q}_{II}^* = 0$, while the output of Parent I as well as that of the subsidiaries, $(\hat{q}_I^*, \hat{\chi}_I^*, \hat{\chi}_{II}^*)$, are positive. The intuition behind this result is that when parents use asymmetric accounting policies, Parent I 's incentive to increase its output in order to alleviate double marginalization is counterbalanced by output reductions of Parent II that seeks to increase the cost of its subsidiary's rival. When the tax rate is below $1/2$, Parent II decreases its output all the way to zero. Thus, Parent I becomes the dominant producer in the Latin (home) market, whereas Parent II becomes the dominant producer in the Greek (external) market. Perhaps not surprisingly, in a SPE in which firms use asymmetric accounting policies, the consolidated profit of Parent II is greater than that of Parent I .

Identifying the market outcomes in a SPE involves simple but tedious calculations – see the Appendix. Proposition 3 summarizes our results. The following notation will be useful in describing the equilibria arising in this setting, as well as in the analysis of the next section. Write

$$h(\tau) := \frac{1 + \tau}{12(1 - \tau)}, \quad l(\tau) := \frac{2 - \tau}{3(1 - \tau)}.$$

For $\tau \in [0, 1/2]$, the functions h and l are both decreasing, and satisfy $h(\tau) < l(\tau)$. We assume that $1/8 \leq s \leq l(\tau)$, since otherwise a pure strategy equilibrium does not exist.

Proposition 3. Assume that $\tau \in [0, 1/2]$ and $s \in [1/8, l(\tau)]$. In a SPE in which Parent I keeps one set of books and Parent II keeps two sets of books, the output of Parent II is $\hat{q}_{II}^* = 0$. Moreover:

(P3.1) If $s > h(\tau)$, then the output of Parent I is

$$\hat{q}_I^* = \frac{2}{3b} \left(1 + \frac{2(2-\tau)}{l(\tau)} \frac{l(\tau) - s}{5 - 7\tau + 24(1-\tau)s} \right),$$

and the price in the Latin market satisfies $0 < \hat{p}^* < p_{VI}^*$, while the subsidiaries' outputs are

$$\begin{aligned} \hat{\chi}_I^* &= \frac{1}{4\beta} \left(1 - \frac{3(1+\tau)}{h(\tau)} \frac{s - h(\tau)}{5 - 7\tau + 24(1-\tau)s} \right) \\ \hat{\chi}_{II}^* &= \frac{1}{2\beta} \left(1 + \frac{1+\tau}{h(\tau)} \frac{s - h(\tau)}{5 - 7\tau + 24(1-\tau)s} \right). \end{aligned}$$

and the price in the Greek market is $\hat{\pi}^* < \pi_{VI}^*$

(P3.2) If $s \leq h(\tau)$, then the price in the Latin market is $\hat{p}^* = 0$, while the subsidiaries' outputs are

$$(\hat{\chi}_I^*, \hat{\chi}_{II}^*) = \left(\frac{1}{4\beta}, \frac{1}{2\beta} \right)$$

and the price in the Greek market is $\hat{\pi}^* = 1/4 < \pi_{VI}^*$.

Moreover, in either case the surpluses in the Latin and Greek markets, \hat{S}^* and $\hat{\Sigma}^*$, respectively, are larger than under vertical integration, i.e., $\hat{S}^* > S_{VI}^*$ and $\hat{\Sigma}^* > \Sigma_{VI}^*$, whereas the consolidated profit of the industry, $\hat{C}_I^* + \hat{C}_{II}^*$, is smaller than under vertical integration, i.e., $\hat{C}_I^* + \hat{C}_{II}^* < 2C_{VI}^*$.

When firms maintain asymmetric accounting policies, Parent II (the firm keeping two sets of books) supplies no output to the Latin market in order to warrant its subsidiary a dominant position in the Greek market. Parent I (the firm keeping one set of books), besides its apparent dominant position in the Latin market, supplies a large output in order to counterbalance its subsidiary disadvantage. The smaller the relative size of the Latin market s , the larger is the output of Parent I: by increasing its output in the Latin market, thus reducing the price, Parent I alleviates double marginalization, and increases the tax bill of its subsidiary's rival. Moreover, the benefits to Parent I of increasing its output are larger the larger is the tax rate τ , and hence \hat{q}_I^* increases with τ .

The equilibrium output (price) in both markets are greater (smaller), and hence the surplus is larger, than under vertical integration. Thus, adopting the ALP to regulate transfer prices improves market efficiency when at least one firm keep two sets of books.

4 The Choice of Accounting Policies

We study parents' choice of accounting policies. In a pure strategy SPE, following accounting policy decisions the parents' payoffs in the space of parameter constellations

$$A := \{(\tau, s) \in [0, 1/2] \times [1/8, \infty] \mid s < l(\tau)\}.$$

are identified by propositions 1-3. (Assuming that $s > 1/8$ and $s < l(\tau)$ is required to fall within the scope of Proposition 3.) Thus, parents' payoffs when they choose to keep either one set of books (action I) or two sets of books (action II) are described in Table I below. In this table, C_I^* and C_{II}^* , are the parents' consolidated profits in the equilibrium arising when both parents keep one set of books, and when both parents keep two sets of books, respectively. These values are calculated using the results of propositions 1 and 2, respectively. Likewise, \hat{C}_I^* (\hat{C}_{II}^*) is the consolidated profit of the parent that keeps one (two) set(s) of books in the equilibrium arising when parents maintain asymmetric accounting policies. These values are calculated using the results of Proposition 3.

G	I	II
I	C_I^*, C_I^*	$\hat{C}_I^*, \hat{C}_{II}^*$
II	$\hat{C}_{II}^*, \hat{C}_I^*$	C_{II}^*, C_{II}^*

Table 1: Parents Choice of Accounting Policies

Thus, at the stage of choosing their accounting policies, anticipating their ensuing output choices and transfer pricing decisions, as well as the output choices of their subsidiaries following these decisions, parents face the two-player two-action symmetric game G described in Table 1. A pure strategy Nash equilibrium (NE henceforth) of G identifies a SPE equilibrium of the dynamic the game played by parents and subsidiaries. The nature of the game G is determined by the sign of the differences

$\hat{C}_{II}^* - C_I^*$ and $C_{II}^* - \hat{C}_I^*$, as shown in the following table.

G	$\hat{C}_{II}^* - C_I^*$	$C_{II}^* - \hat{C}_I^*$
Prisoners' Dilemma	+	+
Chicken	+	−
Coordination	−	+
Cooperation	−	−

Table 2: The Nature of Game G

When G is a *prisoners' dilemma game*, its unique NE involves both firms using the dominant strategy II . When G is a *game of Chicken*, both (II, I) and (I, II) are NE. When G is a *coordination game*, both (I, I) and (II, II) are NE. (Recall that $C_I^* > C_{II}^*$, and hence (I, I) Pareto dominates (II, II) .) Finally, when G is a *cooperation game*, its unique NE involves both firms using the dominant strategy I .

We study the sign of the differences $(\hat{C}_{II}^* - C_I^*, C_{II}^* - \hat{C}_I^*)$ across the space A , and show that all four types of games may arise. To study this issue, it is useful to partition A in four regions:

$$\begin{aligned}
A_1 & : = \{(\tau, s) \in A \mid s > \max\{g(\tau), h(\tau)\}\}, \\
A_2 & : = \{(\tau, s) \in A \mid h(\tau) < s \leq g(\tau), \\
A_3 & : = \{(\tau, s) \in A \mid g(\tau) < s \leq h(\tau). \\
A_4 & : = \{(\tau, s) \in A \mid s \leq \min\{g(\tau), h(\tau)\}\}.
\end{aligned}$$

Figure 1 below shows the graphs of the functions g , h , and l , and identifies these regions. In region A_1 , consolidated profits are identified in propositions 1, (P2.1) and (P3.1). In region A_2 , consolidated profits are identified in propositions 1, (P2.2) and (P3.1). In region A_3 , consolidated profits are identified in propositions 1, (P2.1) and (P3.2). In region A_4 , consolidated profits are identified in propositions 1, (P2.2) and (P3.2). Signing the differences $(\hat{C}_{II}^* - C_I^*, C_{II}^* - \hat{C}_I^*)$ in these regions is simple but tedious – see the Appendix. Our results are summarized in Proposition 4.

Proposition 4. *Assume that $\tau \in [0, 1/2]$ and $s \in [1/8, l(\tau)]$.*

(P4.1) *If $s > \max\{g(\tau), h(\tau)\}$, then all four types of games may arise: G is a coordination game when s is small; for intermediate values of s and τ , G is prisoners'*

positive, and then back to negative, while the sign of $C_{II}^* - \hat{C}_I^*$ switches from positive to negative. Thus, in this region all four types of games arise.

Proposition (P4.2) describes our results for $(\tau, s) \in A_2 \cup A_4$. Across this region $C_{II}^* - \hat{C}_I^* < 0$ and $C_{II}^* - \hat{C}_I^* > 0$. Thus, in this region G is a coordination game.

Proposition (P4.3) describes our results for $(\tau, s) \in A_3$. In A_3 , $\hat{C}_{II}^* - C_I^* < 0$, while $C_{II}^* - \hat{C}_I^*$ is positive (negative) above (below) the green curve. Thus, in this region G is either a coordination game or cooperation game.

Therefore, in region $A_2 \cup A_4 \cup A_3$ only symmetric accounting policies can be sustained by SPE. Moreover, keeping one set of books is sustained by a SPE across the region – uniquely in the subset of A_3 below the green curve. Since $C_I^* > C_{VI}^* > C_{II}^*$ by propositions 1 and 2, i.e., the firms' consolidated profits in the SPE in which both firms keep one set of books are larger than in the SPE in which both firms keep two sets of books, the former SPE is more likely. (Any notion of equilibrium that accounts for the possibilities of non-binding communication, e.g., see Ferreira (1996)'s communication equilibrium, would identify this SPE as the likely outcome.)

It is interesting to note that even though in $A_2 \cup A_4 \cup A_3$ the Latin market is relatively small, a firm has no incentives to change its accounting policy when both the firm and its competitor keep one set of books: even though a firm may become dominant in the relatively larger Greek market by changing its accounting policy (i.e., by keeping two sets of books rather than one), the maximum profits the firm stands to make are those of a Stackelberg leader, which are half of the monopoly profits. However, in these regions the sum of the firms' profits in the Greek market are closed to the monopoly profits. Hence, the gains in profits in the Greek market to be had from such deviation are small, and do not compensate the losses the firm has incurred in the Latin market. (Recall the equilibrium of a subgame in which firms' accounting policies are asymmetric, the firm keeping two sets of books produces zero in the Latin market – see Proposition 3.)

5 Conclusions

We study the consequences of adopting the ALP in imperfectly competitive markets in which firms are vertically separated. Since transfer prices play the dual role of

allocating costs to subsidiaries and determining the tax liabilities in the jurisdictions where firms operate, the firms' incentives to choose their accounting policies are complex. Since the ALP does not restrict the firms internal pricing decisions, we study the accounting policies that arise in equilibrium, and the properties of the ensuing market outcomes. We show that the accounting policies firms adopt depend on the relative sizes of the home and external markets and on the tax rates, and that all configurations of accounting policies may arise in the parameter space.

The choice of accounting policies serves as a precommitment device. We show that relative to vertically integration, competition in the external (home) market softens (intensifies) when parents keep one set of books, while it intensifies when firms keep two sets of books or when firms adopt asymmetric accounting policies.

Our analysis provides an explanation for the mixed empirical evidence on the use of alternative accounting policies. Further, it contributes to understanding the role of transfer prices, and the incentives present the choice of accounting policies. In contrast to most of the literature on this topic (e.g., Schjelderup and Sorgard (1997), Hyde and Choe (2005), Korn and Lengsfeld (2007), Nielsen et al. (2008), Lemus and Moreno (2019)), which takes as given that firms keep one set of books, and focuses on the analysis of the impact of the ALP on tax revenue, we endogenize the choice of the accounting policies, and show conditions under which in equilibrium firms keep one set of book. Indeed, the existence of such conditions provides a rationale for vertical separation whose motivation under quantity competition is not well understood in the absence of frictions such as that created by the adoption of the ALP for tax purposes.

6 Appendix

Proof of Proposition 1. A parent solves the problem $\max_{q \in \mathbb{R}_+} B_I(q)$, where

$$\begin{aligned} B_I(q) &= C(d(q + q_I^*), q, \pi_I(d(q + q_I^*)), \chi_I(d(q + q_I^*))) \\ &= (1 - \tau) \left((1 - b(q + q_I^*))q + \frac{b(q + q_I^*)(3 - 2b(q + q_I^*))}{9\beta} \right). \end{aligned}$$

Since q_I^* must satisfy the first order condition for a solution to this problem, we get

$$q_I^* = \frac{3b + 9\beta}{27b\beta + 8b^2} = \frac{1 - f(s)}{2b}.$$

Hence

$$p_I^* = d(2q_I^*) = f(s).$$

Using equations (4) and (5)

$$\chi_I^* = \chi_I(p_I^*) = \frac{1 - f(s)}{3\beta},$$

and

$$\pi_I^* = \pi_I(p_I^*) = \frac{1 + 2f(s)}{3}.$$

Direct calculation yields

$$C_I^* - C_{VI}^* = (1 - \tau) \frac{2}{9\beta} \frac{4 + 22s + 27s^2}{(8 + 27s)^2} > 0,$$

and

$$S_I^* + \Sigma_I^* - (S_{VI}^* + \Sigma_{VI}^*) = -\frac{2}{9\beta} \frac{20 + 155s + 297s^2}{(8 + 27s)^2} < 0. \quad \square$$

Proof of Proposition 2. A parent solves the problem $\max_{(q,r) \in \mathbb{R}_+ \times \mathbb{R}} B_{II}(q, r)$, where

$$\begin{aligned} B_{II}(q, r) &= C(d(q + q_{II}^*), q, \pi_{II}(d(q + q_{II}^*), r, r_{II}^*), \chi_{II}(d(q + q_{II}^*), r, r_{II}^*)) \\ &= (1 - \tau) (1 - b(q + q_{II}^*)) q \\ &\quad + \frac{1 - \tau}{9\beta} \left(1 + \frac{\tau (1 - b(q + q_{II}^*)) - 2r + r_{II}^*}{1 - \tau} \right) \left(1 - \frac{2\tau (1 - b(q + q_{II}^*)) - r - r_{II}^*}{1 - \tau} \right). \end{aligned}$$

Since (q_{II}^*, r_{II}^*) must satisfy the first order condition for a solution to this problem,

we get

$$(q_{II}^*, r_{II}^*) = \left(\frac{2s + g(\tau)}{6bs}, \frac{\tau(s - g(\tau))}{3s} - \frac{1 - \tau}{5} \right).$$

Hence

$$p_{II}^* = d(2q_{II}^*) = \frac{s - g(\tau)}{3s}.$$

provided $s > g(\tau)$. Using equations (6) and (7)

$$\chi_{II}^* = \chi_{II}(p_{II}^*, r_{II}^*, r_{II}^*) = \frac{2}{5\beta},$$

and

$$\pi_{II}^* = \pi_{II}(p_{II}^*, r_{II}^*, r_{II}^*) = \frac{1}{5}.$$

If $s \leq g(\tau)$, then in equilibrium $p_{II}^* = 0$, and parents' problem becomes $\max_{(q,r) \in \mathbb{R}_+ \times \mathbb{R}} \bar{B}_{II}(r)$, where

$$\begin{aligned} \bar{B}_{II}(r) &= C(0, q, \pi_{II}(0, r, r_{II}^*), \chi_{II}(0, r, r_{II}^*)) \\ &= (1 - \tau) \pi_{II}(0, r, r_{II}^*) \chi_{II}(0, r, r_{II}^*) \\ &= \frac{1 - \tau}{9\beta} \left(1 - \frac{2r - r_{II}^*}{1 - \tau}\right) \left(1 + \frac{r + r_{II}^*}{1 - \tau}\right). \end{aligned}$$

Since r_{II}^* must satisfy the first order condition for a solution to this problem, we get

$$r_{II}^* = -\frac{1 - \tau}{5}.$$

Using again equations (6) and (7) we get we get $\chi_{II}^* = 2/5\beta$ and $\pi_{II}^* = 1/5$.

Since the outputs supplied in both the Latin and Greek markets are larger than under vertical separation, and surplus increases with output on $[0, 1]$, the surpluses in both markets are greater than under vertical integration, i.e., $S_{II}^* > S_{VI}^*$ and $\Sigma_I^* > \Sigma_{VI}^*$. Moreover, consolidated profits decrease with output above the monopoly output (which is equal to $1/2$), and hence consolidated profits are smaller than under vertical integration, i.e., $C_{II}^* < C_{VI}^*$. Of course, these inequalities can be directly verified. \square

Proof of Proposition 3. Let us be given a SPE in which one firm keeps one set of books and the other keeps two sets of book, and denote by $(\hat{\chi}_I^*, \hat{\chi}_{II}^*, \hat{\pi}^*, \hat{q}_I^*, \hat{q}_{II}^*, r_{II}^*, \hat{p}^*)$ the ensuing outputs and prices in the Greek and Latin markets. Then these values satisfy the conditions:

$$\hat{\chi}_I = \max \left\{ \frac{1 - \hat{p} - \beta \hat{\chi}_{II}}{2\beta}, 0 \right\} \quad (8)$$

$$\hat{\chi}_{II} = \max \left\{ \frac{1 - \hat{r}_{II} - (\hat{\pi} - \hat{p})\tau - \beta \hat{\chi}_I}{2\beta}, 0 \right\} \quad (9)$$

$$\hat{\pi} = \delta (\hat{\chi}_I + \hat{\chi}_{II}) \quad (10)$$

$$\hat{p} = d(\hat{q}_I + \hat{q}_{II}) \quad (11)$$

$$\hat{q}_I \in \arg \max_{q \in \mathbb{R}_+} C(d(q + \hat{q}_{II}), q, \hat{\pi}(d(q + \hat{q}_{II}), \hat{r}_{II}), \hat{\chi}_I(d(q + \hat{q}_{II}), \hat{r}_{II})) \quad (12)$$

$$(\hat{q}_{II}, \hat{r}_{II}) \in \arg \max_{(q,r) \in \mathbb{R}_+ \times \mathbb{R}} C(d(q + \hat{q}_I), q, \hat{\pi}(d(q + \hat{q}_I), r), \hat{\chi}_{II}(d(q + \hat{q}_I), r)). \quad (13)$$

We establish a number of preliminary results that will help us proving Proposition 3.

Claim 1. If $\hat{\chi}_I^*, \hat{\chi}_{II}^* > 0$, then

$$\begin{aligned}\hat{\chi}_I^* &= \frac{1 - \hat{p}^*}{3\beta} - \frac{(\hat{p}^* - \hat{r}_{II}^*)}{3\beta(1 - \tau)} \\ \hat{\chi}_{II}^* &= \frac{1 - \hat{p}^*}{3\beta} + \frac{2(\hat{p}^* - \hat{r}_{II}^*)}{3\beta(1 - \tau)}.\end{aligned}$$

and

$$\hat{\pi}^* = \frac{1 + 2\hat{p}^*}{3} - \frac{(\hat{p}^* - \hat{r}_{II}^*)}{3(1 - \tau)} > 0.$$

Proof. If $\hat{\chi}_I^*, \hat{\chi}_{II}^* > 0$, then equations (8) and (9) become the formulae for $\hat{\chi}_I^*$ and $\hat{\chi}_{II}^*$, respectively, given in the claim. Moreover, $\hat{\chi}_I^* > 0$ and $\hat{p}^* \geq 0$ imply

$$0 < 1 - \hat{p}^* - (\hat{p}^* - \hat{r}_{II}^*)/(1 - \tau) \leq 1 - (\hat{p}^* - \hat{r}_{II}^*)/(1 - \tau),$$

and therefore

$$1 - \beta(\hat{\chi}_I^* + \hat{\chi}_{II}^*) = \frac{1}{3} \left(1 + 2\hat{p}^* - \frac{\hat{p}^* - \hat{r}_{II}^*}{1 - \tau} \right) > 0,$$

which using equation (10) establishes the claim. \square

Claim 2. If $\hat{\chi}_{II}^* > 0$, then $\hat{r}_{II}^* \leq \hat{p}^*$ and $\hat{r}_{II}^* \in (-\frac{1}{2}, \frac{1}{4})$.

Proof. Assume that $\hat{\chi}_{II}^* > 0$.

If $\hat{\chi}_I^* > 0$, then $(\hat{\chi}_I^*, \hat{\chi}_{II}^*, \hat{\pi}^*)$ are given by the formulae of Claim 1. Using these values, condition (13) for Parent II 's profit maximization leads to the equation

$$\hat{r}_{II}^* = -\frac{1 - \tau + (1 - 5\tau)\hat{p}^*}{4}.$$

Hence $\tau \in (0, 1/2)$ and $\hat{p}^* \in [0, 1]$ imply

$$\hat{r}_{II}^* < -\frac{1 - \frac{1}{2} + (1 - \frac{5}{2})\hat{p}^*}{4} < -\frac{\frac{1}{2} + (1 - \frac{5}{2})}{4} = \frac{1}{4}$$

and

$$\hat{r}_{II}^* > -\frac{1 - 0 + (1 - \frac{5}{2}(0))\hat{p}^*}{4} = -\frac{1 + \hat{p}^*}{4} > -\frac{1}{2}.$$

Moreover,

$$\hat{r}_{II}^* - \hat{p}^* = -\frac{1 - \tau + (1 - 5\tau)\hat{p}^*}{4} - \hat{p}^* = -\frac{(1 - \tau)(1 + 5\hat{p}^*)}{4} < 0.$$

If $\hat{\chi}_I^* = 0$, then equations (9) and (10) yield

$$(\hat{\chi}_{II}^*, \hat{\pi}^*) = \left(\frac{1 - \hat{r}_{II}^* - (1 - \hat{p}^*)\tau}{2\beta(1 - \tau)}, 1 - \frac{1 - \hat{r}_{II}^* - (1 - \hat{p}^*)\tau}{2(1 - \tau)} \right).$$

Substituting these values and solving (13), we obtain $\hat{r}_{II}^* = -\tau\hat{p}^* \in (-1/2, 0)$, which establishes the claim. \square

Claim 3. $\hat{\chi}_I^*, \hat{\chi}_{II}^*, \hat{q}_I^*, \hat{q}_{II}^* > 0$ does not hold; i.e., no interior SPE exists.

Proof. Assume by way of contradiction that $\hat{\chi}_I^*, \hat{\chi}_{II}^*, \hat{q}_I^*, \hat{q}_{II}^* > 0$. Using the values given in Claim 1 to solve the parents maximization problems (12) and (13) we get

$$\begin{aligned}\hat{q}_I^* &= \frac{1}{2b} \frac{(1-2\tau)s^2 + 2(5-4\tau)s + 12(1-\tau)}{(1-2\tau)s + 18(1-\tau)} \\ \hat{q}_{II}^* &= -\frac{1}{2b} \frac{(1-2\tau)s^2 + 2(5-4\tau)s - 12(1-\tau)}{(1-2\tau)s + 18(1-\tau)} \\ \hat{r}_{II}^* &= -\frac{1}{2} \frac{(1-2\tau)((1-3\tau)s + 12(1-\tau))}{(1-2\tau)s + 18(1-\tau)}.\end{aligned}$$

Substituting these values into equation (11) to obtain \hat{p} , and then into the equation for $\hat{\chi}_I^*$ in Claim 1 we get

$$\hat{\chi}_I^* = -\frac{1}{2\beta} \frac{(1-2\tau)s}{(1-2\tau)s + 18(1-\tau)},$$

which is negative for $\tau \in (0, 1/2)$. This contradiction establishes the claim. \square

Claim 4. If $\hat{q}_{II}^* = 0$, then $\hat{q}_I^* > 0$.

Proof. Assume $\hat{q}_{II}^* = 0$. Then $\hat{p}^* = d(\hat{q}_I^*)$.

Assume that $\hat{r}_{II}^* \leq \hat{p}^* - (1-\tau)(1-\hat{p}^*)$. Then $\hat{\chi}_I^* = 0$, and Parent I 's profit maximization condition (12) implies

$$\hat{q}_I^* = \frac{1}{2b} > 0.$$

Assume that $\hat{p}^* - (1-\tau)(1-\hat{p}^*) < \hat{r}_{II}^* < \hat{p}^* - (1-\tau)(1-\hat{p}^*)/2$. Then $\hat{\chi}_1^*, \hat{\chi}_2^* > 0$ and condition (12) yields

$$\hat{q}_I^* = \frac{1}{2b} \frac{9(1-\tau)^2 + (5(1-2\tau) + 3\tau^2)s + (1+\tau)s\hat{r}_{II}^*}{9(1-\tau)^2 + (1-2\tau)(2-\tau)s} > 0.$$

Since $\hat{r}_{II}^* > -\frac{1}{2}$ by Claim 2,

$$\begin{aligned}\hat{q}_I^* &> \frac{1}{2b} \frac{9(1-\tau)^2 + (5(1-2\tau) + 3\tau^2)s + (1+\tau)s(-\frac{1}{2})}{9(1-\tau)^2 + (1-2\tau)(2-\tau)s} \\ &= \frac{1}{2b} \frac{18(1-\tau)^2s + 3(1-2\tau)(3-\tau)}{18(1-\tau)^2s + 2(1-2\tau)(2-\tau)} > 0.\end{aligned}$$

Finally, assume that $\hat{r}_{II}^* > \hat{p}^* - (1 - \tau)(1 - \hat{p}^*)/2$. Then $\hat{\chi}_{II}^* = 0$ and Parent I 's profit maximization condition (12) yields

$$\hat{q}_I^* = \frac{2s + 1}{b(4s + 1)} > 0. \quad \square$$

Claim 5. $\hat{q}_I^* > 0$.

Proof. Assume by way of contradiction that $\hat{q}_I^* = 0$. Then $\hat{q}_{II}^* > 0$ by Claim 4, and $\hat{p}^* = d(\hat{q}_{II}^*)$ by equation (11).

Assume that $\hat{r}_{II}^* < \hat{p}^* - (1 - \tau)(1 - \hat{p}^*)$. Then $\hat{\chi}_I^* = 0$, and therefore

$$\hat{\chi}_{II}^* = \frac{1 - \hat{p}^*}{2\beta} + \frac{\hat{p}^* - \hat{r}_{II}^*}{2\beta(1 - \tau)},$$

by equation (9). Since $\hat{q}_{II}^* > 0$, then Parent II 's profit maximization condition (13) yields the equations

$$\begin{aligned} \hat{q}_{II}^* &= \frac{2\beta(\tau - 1)^2 - b\tau(\hat{r}_{II}^* - \tau)}{b(b\tau^2 + 4\beta(1 - \tau)^2)} \\ \hat{r}_{II}^* &= \tau(1 - b\hat{q}_{II}^*). \end{aligned}$$

Solving this system of equations, we get $\hat{q}_{II}^* = 1/(2b)$ and $\hat{r}_{II}^* = \tau/2$. Substituting these values into the equation for $\hat{\chi}_{II}^*$ above, we get $\hat{\chi}_{II}^* = 1/(2\beta)$. For these values, however, Parent I 's profit maximization condition (12) is

$$\hat{q}_I^* = \frac{1}{4b} > 0,$$

contradicting that $\hat{q}_I^* = 0$.

Assume that $\hat{p}^* - (1 - \tau)(1 - \hat{p}^*) < \hat{r}_{II}^* < \hat{p}^* - (1 - \tau)(1 - \hat{p}^*)/2$. Then $\hat{\chi}_I^*, \hat{\chi}_{II}^* > 0$. Using that formulae in Claim 1 and noticing that $\hat{q}_{II}^* > 0$ we can write Parent II 's profit maximization conditions as

$$\begin{aligned} \hat{q}_{II}^* &= \frac{9(1 - \tau)^2 s + 5(1 - 2\tau) + 3\tau^2 - 3(3 - 5\tau) + (1 - 5\tau)\hat{r}_{II}^*}{2b(9(1 - \tau)^2 s - (1 - 2\tau)(1 + \tau))}, \\ \hat{r}_{II}^* &= -\frac{1 - 3\tau}{2} + \frac{b}{4}(1 - 5\tau)\hat{q}_{II}^*, \end{aligned}$$

Solving this system of equations, we get

$$(\hat{q}_{II}^*, \hat{r}_{II}^*) = \left(\frac{2(1 - 2s)}{b(1 - 8s)}, \frac{3s - \tau(7s + 1)}{1 - 8s} \right).$$

Using again the equations for $\hat{\chi}_I^*, \hat{\chi}_{II}^*$ we get

$$(\hat{\chi}_I^*, \hat{\chi}_{II}^*) = \left(\frac{1+s}{\beta(1-8s)}, -\frac{6}{\beta(1-8s)} \right).$$

Hence either $\hat{\chi}_1^* < 0$ or $\hat{\chi}_{II}^* < 0$, which contradicts that $\hat{\chi}_1^*, \hat{\chi}_{II}^* > 0$.

Finally, assume that $\hat{r}_{II}^* > \hat{p}^* - (1-\tau)(1-\hat{p}^*)/2$. Then $\hat{\chi}_{II}^* = 0$, and therefore $\hat{\chi}_I^* = (1-\hat{p}^*)/2\beta$ by equation (8). Since $\hat{q}_{II}^* > 0$, Parent II 's profit maximization condition (13) yields $\hat{q}_{II}^* = 1/2b$. Then Parent I 's profit maximization condition (12) implies

$$\hat{q}_I^* = \frac{1}{2b} \frac{1+2s}{1+4s} > 0,$$

contradicting that $\hat{q}_I^* = 0$. \square

Claim 6. $\hat{\chi}_I^* > 0$.

Proof. Assume by way of contradiction that $\hat{\chi}_I^* = 0$. Then equation (9) yields

$$\hat{\chi}_{II}^* = \frac{1-\hat{p}^*}{2\beta} + \frac{\hat{p}^* - \hat{r}_{II}^*}{2\beta(1-\tau)} > 0,$$

where the inequality holds since $\hat{r}_{II}^* \leq \hat{p}^*$ by Claim 2. Since $\hat{q}_I^* > 0$ by Claim 5, the first-order condition for Parent 1's profit maximization yields

$$\hat{q}_I^* = \frac{1 - b\hat{q}_{II}^*}{2b},$$

and the first-order conditions for Parent II 's profit maximization yields the system

$$\begin{aligned} \hat{q}_{II}^* &= \max \left(0, \frac{(2(1-\tau)^2 s + \tau^2)(1 - b\hat{q}_I^*) - \tau\hat{r}_{II}^*}{b(4s(1-\tau)^2 + \tau^2)} \right), \\ \hat{r}_{II}^* &= \tau(1 - b(\hat{q}_I^* + \hat{q}_{II}^*)). \end{aligned}$$

Solving this system of equations, we get $(\hat{q}_I^*, \hat{q}_{II}^*, \hat{r}_{II}^*) = (1/3b, 1/3b, \tau/3)$. Hence $\hat{p}^* = 1/3$. Substituting these values into equation (8) yields

$$\hat{\chi}_I^* = \frac{1}{12\beta} > 0,$$

contradicting that $\hat{\chi}_I^* = 0$. \square

Claim 7. $\hat{\chi}_{II}^* > 0$.

Proof. Assume by way of contradiction that $\hat{\chi}_{II}^* = 0$. Then

$$\hat{\chi}_I^* = \frac{1-\hat{p}^*}{2\beta} > 0,$$

by equation (8). Since $\hat{q}_I^* > 0$ by Claim 5, Parent I 's profit maximization condition (12) yields

$$\hat{q}_I^* = \frac{(2s+1)(1-b\hat{q}_{II}^*)}{b(4s+1)}.$$

And since $\hat{\chi}_{II}^* = 0$, then the consolidated profit of Parent II is independent of r_{II}^* , and profit maximization condition (13) yields

$$\hat{q}_{II}^* = \max \left(0, \frac{1}{2b} (1 - b\hat{q}_I^*) \right).$$

Solving the two equations above for $(\hat{q}_I^*, \hat{q}_{II}^*)$ we get

$$(\hat{q}_I^*, \hat{q}_{II}^*) = \left(\frac{2s+1}{b(6s+1)}, \frac{2s}{b(6s+1)} \right).$$

Using the values for $(\hat{q}_I^*, \hat{q}_{II}^*)$ we readily calculate

$$\begin{aligned} \hat{p}^* &= \frac{2s}{6s+1} \\ \hat{\chi}_I^* &= \frac{4s+1}{2\beta(1+6s)}. \end{aligned}$$

Using the values of \hat{q}_I^* and $\hat{\chi}_I^*$ to calculate Parent II 's consolidated profit we calculate the first order condition for a solution to this firm's problem as

$$\begin{aligned} \hat{q}_{II}^* &= \frac{36(1-\tau)^2 s^2 + 2(5\tau^2 + 13\tau - 10)s - (1-\tau)(2-\tau) + (6s+1)(1-5\tau)\hat{r}_{II}^*}{2b(9(1-\tau)^2 s^2 - (1-2\tau)(1+\tau)s)(6s+1)}, \\ \hat{r}_{II}^* &= -\frac{2(5-13\tau)s + (1-\tau) + b(6s+1)(1-5\tau)\hat{q}_{II}^*}{4(6s+1)}. \end{aligned}$$

Solving this system, we get

$$\hat{q}_{II}^* = \frac{(10s+1) - 16s^2}{b(6s+1)(1-8s)} \neq \frac{2s}{b(6s+1)}$$

for all $s > 0$, which leads to a contradiction. \square

Claim 8. $\hat{q}_{II}^* = 0$.

Proof. Since $\hat{q}_I^*, \hat{\chi}_{II}^*, \hat{\chi}_{II}^* > 0$, by claims 5, 6 and 7, Claim 3 implies $\hat{q}_{II}^* = 0$. \square

We complete the proof of Proposition 3. Since $\chi_I^*, \chi_{II}^* > 0$ by claims 6 and 7, we can use the equations in Claim 1 to calculate the consolidated profits of Parent I ,

and derive the first order condition for an interior solution to its profit maximization problem. Solving this equation we get

$$\hat{q}_I^* = \frac{9(1-\tau)^2 s + 5(1-2\tau) + 3\tau^2 + (1+\tau)r_{II}^*}{2b(9(1-\tau)^2 s + (1-2\tau)(2-\tau))}.$$

Likewise, using the equations in Claim 1 and noting that $\hat{q}_{II}^* = 0$ by Claim 8, we calculate the consolidated profits of Parent II , and identify the first order condition for an interior solution to its profit maximization problem. Solving this equation we get

$$\hat{r}_{II}^* = -\frac{1-3\tau}{2} + \frac{b}{4}(1-5\tau)\hat{q}_I^*. \quad (14)$$

Solving the system form by these two equations we get

$$\hat{q}_I^* = \frac{1}{b} \left(1 - \frac{12(s-h(\tau))(2-\tau)}{5-7\tau+24(1-\tau)s} \right) \quad (15)$$

$$\hat{r}_{II}^* = -\frac{(1-\tau)}{4} - \frac{3(s-h(\tau))(2-\tau)(1-5\tau)}{5-7\tau+24(1-\tau)s}. \quad (16)$$

Hence the price in the Latin market is

$$\hat{p}^* = \max\{1 - b\hat{q}_I^*, 0\} = \max\left\{ \frac{12(s-h(\tau))(2-\tau)}{5-7\tau+24(1-\tau)s}, 0 \right\}. \quad (17)$$

Assume that $s > h(\tau)$. Then the value for \hat{p}^* in (17) is positive. Substituting the values of \hat{r}_{II}^* and \hat{p}^* in the formulae of Claim 1 we get

$$\begin{aligned} \hat{\chi}_I^* &= \frac{1}{4\beta} \left(1 - \frac{1}{h(\tau)} \frac{3(1+\tau)(s-h(\tau))}{5-7\tau+24(1-\tau)s} \right) \\ \hat{\chi}_{II}^* &= \frac{1}{2\beta} \left(1 + \frac{1}{h(\tau)} \frac{(1+\tau)(s-h(\tau))}{5-7\tau+24(1-\tau)s} \right). \end{aligned}$$

Hence the price in the Greek market is

$$\hat{\pi}^* = 1 - \beta(\hat{\chi}_I^* + \hat{\chi}_{II}^*) = \frac{1}{4} \left(1 + \frac{1}{h(\tau)} \frac{(1+\tau)(s-h(\tau))}{5-7\tau+24(1-\tau)s} \right) > 0.$$

In order to verify the values for $(\hat{q}_I^*, \hat{r}_{II}^*, \hat{p}^*, \hat{\chi}_I^*, \hat{\chi}_{II}^*, \hat{\pi}^*)$ given by these formulae form an equilibrium we must show that $\hat{q}_{II}^* = 0$ maximizes Parent II 's consolidated profit. Given the value of \hat{q}_I^* above and the formulae of Claim 1 for the subsidiaries' outputs, we calculate the consolidated profit of Parent II . Taking derivatives with respect to Parent II 's output and transfer price of obtain yields the system of first-order

conditions

$$\begin{aligned}\hat{q}_{II}^* &= \frac{3(1-\tau)(36(1-\tau)^2 s^2 - (27 - (32 + 11\tau)\tau)s + \tau(\frac{29}{3} - \tau)) - 8}{2b(9(1-\tau)^2 s - (1-2\tau)(1+\tau))(24(1-\tau)s + (5-7\tau))} \\ &\quad + \frac{(1-5\tau)\hat{r}_{II}^*}{2b(9(1-\tau)^2 s - (1-2\tau)(1+\tau))} \\ \hat{r}_{II}^* &= -\frac{3(3-7\tau)(1-\tau)s + 1-\tau(2-3\tau)}{24(1-\tau)s + (5-7\tau)} + \frac{b}{4}(1-5\tau)\hat{q}_{II}^*.\end{aligned}$$

Solving this system, we get

$$\hat{q}_{II}^* = \frac{1}{b} \frac{4(12(1-\tau)s^2 - 2(5-4\tau)s - (1-2\tau))}{(8s-1)(24(1-\tau)s + (5-7\tau))}.$$

Since $s > 1/8$ by assumption, the denominator of this expression is positive. We show that the numerator is negative, thus establishing that the solution to Parent II 's problem involves $\hat{q}_{II} = 0$. Since $\tau \in [0, 1/2]$ and $s < l(\tau) = (2-\tau)/3(1-\tau)$ by assumption,

$$\begin{aligned}12(1-\tau)s^2 - 2(5-4\tau)s - (1-2\tau) &< (12(1-\tau)s - 2(5-4\tau))s \\ &< \left(12(1-\tau)\frac{2-\tau}{3(1-\tau)} - (10-8\tau)\right)s \\ &= -4\left(\frac{1}{2} - \tau\right)s \\ &< 0.\end{aligned}$$

Now assume that $s \leq h(\tau)$. Then in equilibrium $\hat{p}^* = 0$, and therefore $\hat{q}_I^* \geq 1/b$. Then the first-order condition for Parent II 's profit maximization yields

$$\hat{r}_{II}^* = -\frac{1-\tau}{4},$$

and therefore, using the formulae in Claim 1 we get

$$(\chi_{II}^*, \chi_{II}^*) = \left(\frac{1}{4b}, \frac{1}{2b}\right).$$

The equilibrium price in the Greek markets is

$$\hat{\pi}^* = \frac{1}{4}.$$

In order for $\hat{q}_{II}^* = 0$ to maximize the consolidated profits of Parent II taking as given $\hat{q}_I^* = \frac{1}{b}$, the system defined the first-order conditions that interior solution must

satisfy is

$$\begin{aligned}\hat{q}_{II} &= -\frac{((1-\tau)(2-\tau) + (1-5\tau)\hat{r}_{II}^*)s}{2\beta 9(1-\tau)^2 s - (1-2\tau)(1+\tau)} \\ \hat{r}_{II} &= -\frac{1-\tau}{4} + \frac{b}{4}(1-5\tau)\hat{q}_{II}^*,\end{aligned}$$

which solution involves setting,

$$\hat{q}_{II} = \frac{s}{(1-8s)\beta},$$

which is negative since $s < 1/8$. Hence $\hat{q}_{II}^* = 0$.

Verifying the inequalities relating the surpluses generated in this SPE to those under vertical integration involves direct calculation. Assume that $s > h(\tau)$. Then

$$\hat{q}_I^* + \hat{q}_{II}^* = \hat{q}_I^* = \frac{2}{3b} \left(1 + \frac{2(2-\tau)}{l(\tau)} \frac{l(\tau) - s}{5 - 7\tau + 24(1-\tau)s} \right) > \frac{2}{3b} = 2q_{VI}^*,$$

and

$$\hat{\chi}_I^* + \hat{\chi}_{II}^* = \frac{2}{3\beta} + \frac{1}{3\beta} \frac{(2-\tau) - 3(1-\tau)s}{(5-7\tau) + 24(1-\tau)s} > \frac{2}{3\beta} = 2\chi_{VI}^*.$$

Assume $s \leq h(\tau)$. Then

$$\hat{q}_I^* + \hat{q}_{II}^* = \hat{q}_I^* = \frac{1}{b} > \frac{2}{3b} = 2q_{VI}^*,$$

and

$$\hat{\chi}_I^* + \hat{\chi}_{II}^* = \frac{1}{4\beta} + \frac{1}{2\beta} > \frac{2}{3\beta} = 2\chi_{VI}^*.$$

Hence in either case $\hat{S}^* > S_{VI}^*$ and $\Sigma^* > \Sigma_{VI}^*$.

Likewise, verifying the inequalities relating the industry's consolidated profits to those under vertical integration involves direct calculations. Assume that $s > h(\tau)$. Then

$$\begin{aligned}\hat{C}_I^* + \hat{C}_{II}^* &= \frac{2(1-\tau)}{9b} + \frac{2(1-\tau)}{9\beta} \\ &\quad - \frac{(1-\tau)(2-\tau-3s(1-\tau))}{9bs} \frac{48(1-\tau)s^2 + (73-65\tau)s + 7-8\tau}{((5-7\tau) + 24(1-\tau)s)^2} \\ &< \frac{2(1-\tau)}{9b} + \frac{2(1-\tau)}{9\beta} \\ &= 2C_{VI}^*.\end{aligned}$$

Assume that $s \leq h(\tau)$. Then

$$\hat{C}_I^* + \hat{C}_{II}^* = \frac{1-\tau}{\beta} \frac{1}{16} + \frac{1-\tau}{\beta} \frac{1}{8} = \left(\frac{27}{32} \right) \frac{2(1-\tau)}{9\beta} < \frac{2(1-\tau)}{9b} + \frac{2(1-\tau)}{9\beta} = 2C_{VI}^*. \quad \square$$

Proof of Proposition 4. Using the results of Proposition 1, we calculate each firm's consolidated profit when both firms keep one set of books, to get

$$C_I^* = \frac{1-\tau}{\beta} \left(\frac{f(s)(1-f(s))s}{2} + \frac{(1+2f(s))(1-f(s))}{9} \right). \quad (18)$$

Likewise, using the results of Proposition 2 we calculate each firm's consolidated profits when both firms keep two sets of books, to get

$$C_{II}^* = \frac{1-\tau}{\beta} \left(\frac{(s-g(\tau))(2s+g(\tau))}{18s} + \frac{2}{25} \right) \quad (19)$$

if $s > g(\tau)$, and

$$C_{II}^* = \frac{1-\tau}{\beta} \left(\frac{2}{25} \right) \quad (20)$$

if $s \leq g(\tau)$. Finally, using the results of Proposition 3 we calculate the each firms' consolidated profit when they use asymmetric accounting policies. Define the functions

$$\begin{aligned} n(\tau, s) &= \frac{2(2-\tau)(s-l(\tau))}{l(\tau)(5-7\tau+24(1-\tau)s)} \\ r(\tau, s) &= \frac{(1+\tau)(s-h(\tau))}{h(\tau)(5-7\tau+24(1-\tau)s)}. \end{aligned}$$

The consolidated profit of the firm keeping one set of books is

$$\hat{C}_I^* = \frac{1-\tau}{\beta} \left(\frac{2s}{9} (1+2n(\tau, s))(1-n(\tau, s)) + \frac{1}{16} (1+r(\tau, s))(1-3r(\tau, s)) \right) \quad (21)$$

if $s > h(\tau)$, and it is

$$\hat{C}_I^* = \frac{1-\tau}{\beta} \frac{1}{16} \quad (22)$$

if $s \leq h(\tau)$. The consolidated profit of the firm keeping two sets of books is

$$\hat{C}_{II}^* = \left(\frac{1-\tau}{\beta} \right) \frac{1}{8} (1+r(\tau, s))^2 \quad (23)$$

if $s > h(\tau)$, and it is

$$\hat{C}_{II}^* = \frac{1-\tau}{\beta} \frac{1}{8} \quad (24)$$

if $s \leq h(\tau)$. Hence, calculating the consolidated profits in each region involves the formulae described in the table below.

G	\hat{C}_{II}^*	C_I^*	C_{II}^*	\hat{C}_I^*
A_1	(23)	(18)	(19)	(21)
A_2	(23)	(18)	(20)	(21)
A_3	(24)	(18)	(19)	(22)
A_4	(24)	(18)	(20)	(22)

Table 2: Consolidated Profit Formulae

The red parabola in Figure 2 shows the locus of points in A for which $\hat{C}_{II}^* - C_I^* = 0$, where \hat{C}_{II}^* and C_I^* are calculated using equations (23) and (18), respectively, which are the relevant formulae in regions A_1 and A_2 . Inside (outside) the red parabola $\hat{C}_{II}^* - C_I^*$ is positive (negative). Thus, $\hat{C}_{II}^* - C_I^*$ is negative in A_2 and takes positive and negative values in A_1 .

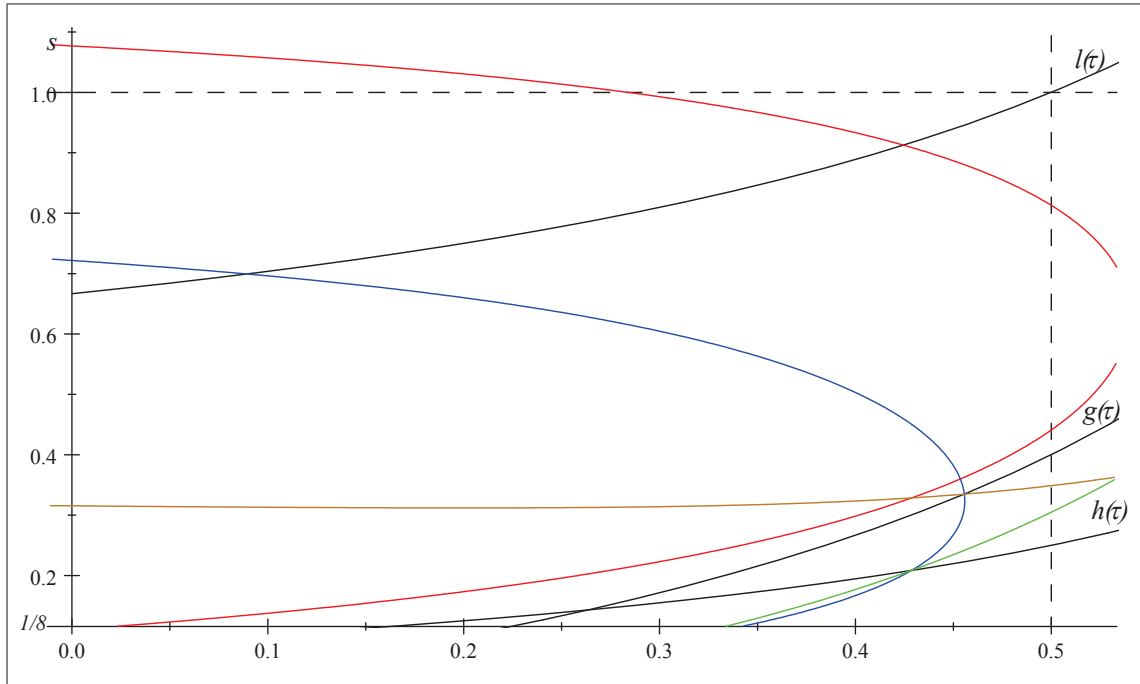


Figure 2: Profit Comparisons

Using equation (24) to calculate \hat{C}_{II}^* we get

$$\hat{C}_{II}^* - C_I^* = -\frac{1 - \tau}{\beta} \frac{3(216s^2 + 117s + 16)s}{8(27s + 8)^2} < 0.$$

Hence $\hat{C}_{II}^* - C_I^*$ is negative in $A_3 \cup A_4$.

The blue parabola in Figure 2 shows the locus of points in A for which $C_{II}^* - \hat{C}_I^* = 0$, where C_{II}^* and \hat{C}_I^* are calculated using equations (19) and (21), respectively, which are the relevant formulae in region A_1 . Inside (outside) the blue parabola $C_{II}^* - \hat{C}_I^*$ is positive (negative). Thus, $C_{II}^* - \hat{C}_I^*$ takes positive and negative values in A_1 .

The brown curve in Figure 2 shows the locus of points in A for which $C_{II}^* - \hat{C}_I^* = 0$, where C_{II}^* and \hat{C}_I^* are calculated using equations (20) and (21), respectively, which are relevant formulae in region A_2 . Above (below) the brown curve $C_{II}^* - \hat{C}_I^*$ is negative (positive). Since A_2 is below the brown curve, $C_{II}^* - \hat{C}_I^*$ is positive in A_2 .

The green curve in Figure 2 shows the locus of points in A for which $C_{II}^* - \hat{C}_I^* = 0$, where C_{II}^* and \hat{C}_I^* are calculated using equations (19) and (24), which are the relevant formulae in region A_3 . Above (below) the green curve $C_{II}^* - \hat{C}_I^*$ is positive (negative). Thus, $C_{II}^* - \hat{C}_I^*$ takes positive and negative values in A_3 .

Finally, using (20) and to calculate C_{II}^* and (22) to calculate $C_{II}^* - \hat{C}_I^*$ we get

$$C_{II}^* - \hat{C}_I^* = \frac{1 - \tau}{\beta} \left(\frac{2}{25} - \frac{1}{16} \right) > 0.$$

Thus, $C_{II}^* - \hat{C}_I^*$ is positive in A_4 .

Table 3 summarizes these results.

G	$\hat{C}_{II}^* - C_I^*$	$C_{II}^* - \hat{C}_I^*$
A_1	+/- (Inside/outside the red parabola)	+/- (Inside/outside the blue parabola)
A_2	-	+
A_3	-	+/- (above/below the green curve)
A_4	-	+

Table 3: Signs of Payoffs Differences in G

The results in Proposition 4 are directly implied by the signs given in Table 3. \square

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